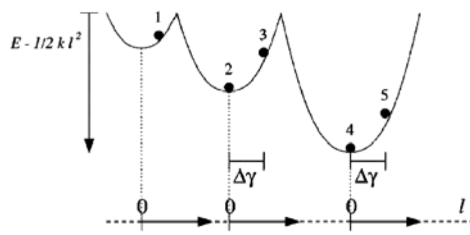
Aging and rheology in soft materials

Fielding, Sollich, and Cates
J. Rheol. 2000

Presented by

Randy H. Ewoldt
Summer Reading Group Presentation
June 27, 2006



Context

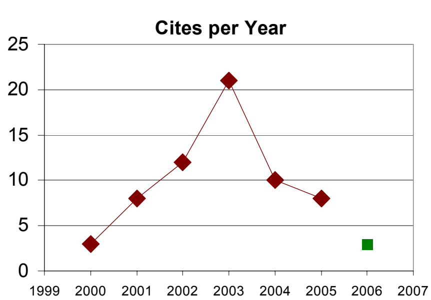
- Prior work
  - Struik, 1978 (book): Aging in polymer glasses
    - Introduced "effective" relaxation time $\tau(t_w)$ to collapse data
    - SGR model is more general
  - Bouchaud, 1992 (272 cites)
    - Introduced hopping (trap) model for disordered (complex) systems (glassy systems)
    - 80% of SGR model is Bouchaud's model (in my humble opinion)
  - Sollich et al., 1997 (165 cites)
    - PRL article, introduced basics of SGR model
  - Sollich, 1998 (136 cites)
    - Phys. Rev. E; describe SGR model in detail

Context

- This work: details of aging in SGR model
- Later work
  - Trap models applied to other areas: fragile glasses, granular materials, pinning of extended defects such as domain walls, etc. (Bertin & Bouchaud, Phys. Rev. E, 2003)
  - Cates & Sollich, JoR, 2004, "…generalizations of the SGR model that combine its nontrivial aging and yield properties with a tensorial structure…"
  - Some results confirmed by experiments (Abou, Bonn, and Meunier, JoR, 2003)

Citation History

65 total cites (Web of Science, June 26, 2006)

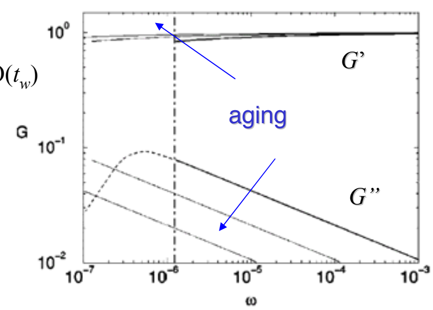


Field: Source Title	Record Count	% of 65	Bar Chart
PHYSICAL REVIEW E	15	23.1 %	
JOURNAL OF RHEOLOGY	10	15.4 %	
PHYSICAL REVIEW LETTERS	9	13.8 %	
JOURNAL OF PHYSICS-CONDENSED MATTER	5	7.7 %	
EUROPEAN PHYSICAL JOURNAL E	3	4.6 %	
MACROMOLECULES	3	4.6 %	
FARADAY DISCUSSIONS	2	3.1 %	
JOURNAL OF CHEMICAL PHYSICS	2	3.1 %	
JOURNAL OF PHYSICS A-MATHEMATICAL AND GENERAL	2	3.1 %	
RHEOLOGICA ACTA	2	3.1 %	

(12 Source Title value(s) outside display options.)

Soft Glassy Materials (SGMs)

- SGM Definition
  - ❑ Material with structural disorder and metastability
  - ❑ e.g. wet foams, emulsions, pastes, and slurries
    - metastable: a state of pseudo-equilibrium having higher free energy than the true equilibrium state
- Typical behavior
  - ❑ Extremely long relaxation time
  - ❑ Aging
    - Relaxation time grows
    - Peak not accessible if $\tau \sim O(t_w)$



Aging

- Time translation invariance (TTI)
  - ❑ Same properties (or results) if test is performed at a different time
  - ❑ Violated by aging or transients
  - ❑ Article develops formal framework of rheology without TTI
- Aging definition
  - ❑ Sample ages if the long time response does not converge for $t_w \to \infty$
  - ❑ Formally: ages if at least one response function violates Eq. 18

$$\lim_{t \to \infty} G(t - t_w, t_w) = \lim_{\Delta t \to \infty} \lim_{t_w \to \infty} G(\Delta t, t_w)$$

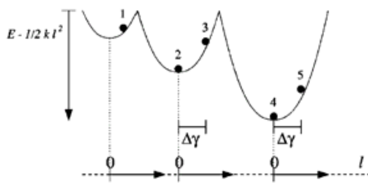
- Transients
  - ❑ Any violation of TTI that is not aging
- Thixotropy - the word is never mentioned!
  - ❑ I believe article's definition of aging is a subset of Barnes' definition of thixotropy
  - ❑ Thixotropy may include the transients as they are defined here
    - e.g. viscosity overshoot

SGR Model

Assumptions

- Scalar model relating shear stress to shear strain
- Mesoscopic, heterogeneous elements
- Continuous variables of stress and strain apply
- Locally affine deformation

  $l = \gamma$



Causality

- Stress response is linear elastic for each element
- Elements have a probability of "yielding" (function of strain l and effective temp x)
- Yielding resets element to *local* strain equilibrium (l=0), at new trap depth E
- New depth probability determined from previous distribution

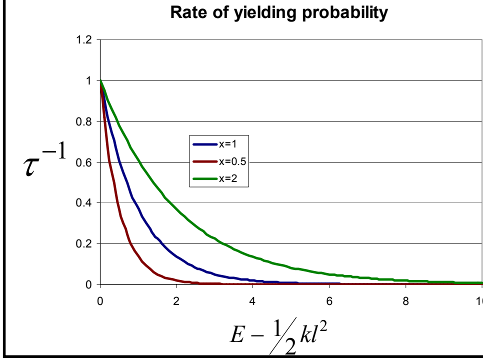
Q: Under continuous macroscopic straining, what is shape of local stress with time?

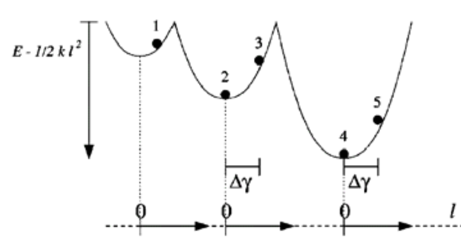
A: sawtooth

SGR Model

- From Bouchaud, with two modifications
  1. Thermal temp replaced with "effective temp" x
  2. Introduce strain into trap model
- Exponential distribution of relaxation times
  - Strain lowers energy barrier

$$\tau = \tau_0 \exp\left[(E - \tfrac{1}{2}kl^2)/x\right]$$





SGR Model – constitutive equations

stress ↔ strain

$$\sigma(t) = \gamma(t)G_0(Z(t,0)) + \int_0^t [\gamma(t)-\gamma(t')]Y(t')G_\rho(Z(t,t'))dt'$$

~ strain rate

conservation of probability

$$1 = G_0(Z(t,0)) + \int_0^t Y(t')G_\rho(Z(t,t'))dt'.$$

units?

Effective time interval (reduces to *t-t'* in limit of small strains)

$$Z(t,t') = \int_{t'}^{t} \exp([\gamma(t'')-\gamma(t')]^2/2x)\,dt''$$

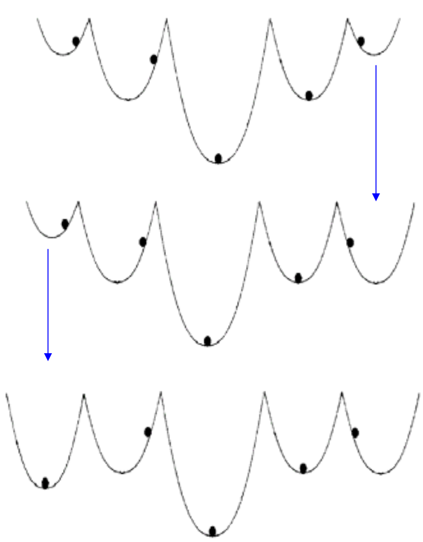
$$G_\rho(Z) = \int_0^\infty \rho(E)\exp(-Ze^{-E/x})dE.$$

distribution of trap depths

Proportion which survive until *t* (which yielded at *t'*)

$G_0(Z) =$ Same as above, but with initial distribution

Cause of Aging for $x < 1$



Q: Which trap(s) most likely to yield first?

A: Shallow traps

Q: How is new trap depth determined?

A: New depth assigned by probability distribution, based on previous distribution of depths

Q: What is most probable new depth?

A: Mean of previous depth distribution

	$x < 1$	$x = 1$	$1 < x$
$Y=$	$\frac{t^{x-1}}{x\Gamma(x)\Gamma(1-x)}$	$\frac{1}{\ln(t)}$	$\frac{x-1}{x}$

Linearity with SGR model

- Linearity requirement
  - Local strains remain small
  - Effects of strain on the effective time interval $Z(t,t')$ are small

- Consequences of linearity
  - Effective time interval $Z(t,t') \rightarrow t - t'$
  - Noise induced yielding

$$\tfrac{1}{2} k l^2 \ll E$$

  - Internal dynamics independent of imposed deformation
  - Lifetimes are independent of γ

$$\tau = \tau_0 \exp\left[(E - \tfrac{1}{2} k l^2)/x\right] \;\rightarrow\; \tau = \tau_0 \exp\left[E/x\right]$$

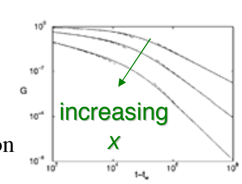
Step Strain

- In general, from Eq. 34

$$G(t - t_w, t_w) = 1 - \int_{t_w}^{t} Y(t') G_\rho(Z(t,t'))dt'$$

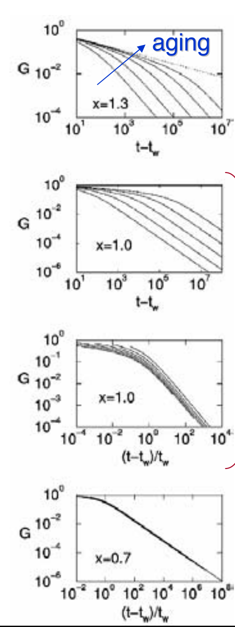
$Z \rightarrow t\text{-}t'$ in linear regime

- Linear
  - Increased x hastens relaxation
  - Aging slows relaxation
  - For $x<1$, data collapses with $t - t'/t_w$ scaling "major part of decay occurs on timescale of order t_w iteslf"



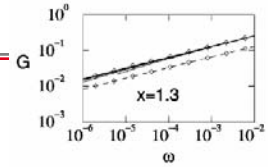
Linear response, t_w constant

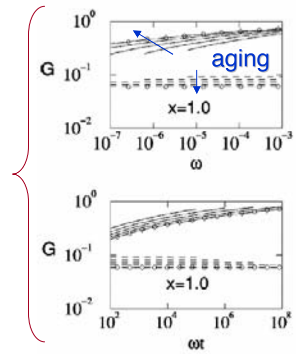
- Nonlinear G v. time
  - Same G_0
  - Faster relaxation (strain hastens yield events through Z)



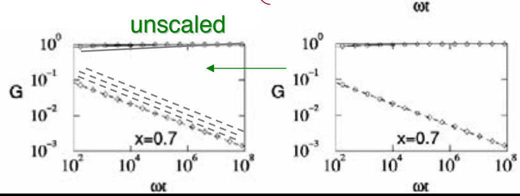
Oscillatory Shear

- Only consider linear oscillation
- Controlled strain and stress give same results
- Applicable only for “low frequencies”
  - $\omega\tau_0 << 1$
    (they set $\tau_0=1$, so often specified as $\omega << 1$)
- For $x<1$
  - Ages such that G' increases and G'' decreases with time
  - data collapses with ωt_w scaling
    (scaling in with t_w similar to step strain)



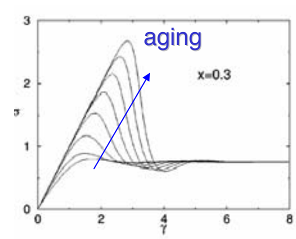


	$1 < x < 2$	$2 < x < 3$	$3 < x$
$G' \sim$	ω^{x-1}		ω^2
$G'' \sim$	ω^{x-1}	ω	



Step Strain Rate

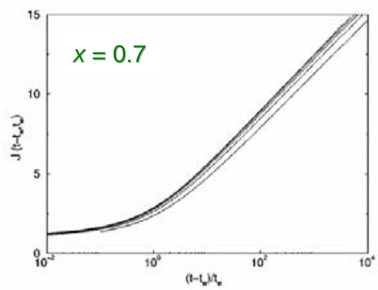
- All results applicable only at “low rates”
  - $\dot{\gamma}\tau_0 << 1$
    (they set $\tau_0=1$, so often specified as $\dot{\gamma} << 1$)
- Linear – uninteresting (limited to small strains)
  - An explicit dependence on t_w resides only in subdominant corrections
  - “Thus, linear shear startup is not a good experimental test of aging or slow transient effects”
  - Purely elastic, anomalous power law, Newtonian (with increasing x)
- Nonlinear
  - Forced rate immobilizes aging
    - even deepest traps eventually yield
    - yield times obey power-law distribution (rather than exponential in E)
  - Overshoot stress why?
    - $\Delta\sigma \sim \ln(t_w)$
    - is undershoot physical?



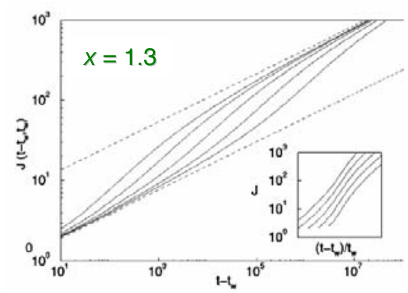
	x < 1	$1 < x < 2$	$2 < x$
$\sigma_{ss} \sim$		$\dot{\gamma}^{x-1}$	$\dot{\gamma}$
$\sigma_{ss} - \sigma_y \sim$	$\dot{\gamma}^{1-x}$	aging interrupted by flow	

Step Stress - linear

- For x<1
  - Aging (weak)
    - ~ $\ln((t-t_w)/t_w)$
    - ~ $\ln(t-t_w) - \ln(t_w)$
  - Strain must increase with time, but at a rate that could tend to zero at long times → yield
- For x>1
  - No “aging”
    - Long time response not affected by t_w
  - Transient affected by t_w
    - So *I* would say the transient experiences aging



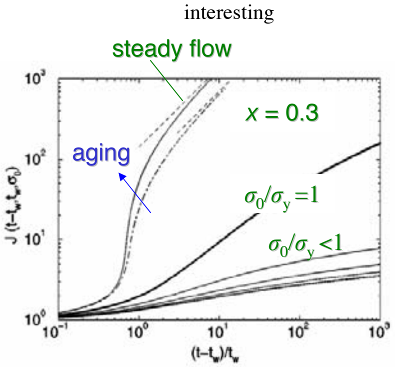
weak correction
for $t >> t_w$

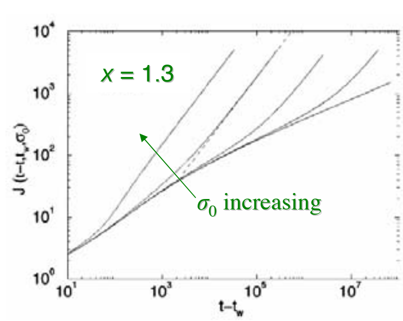


transient affected by t_w

Step Stress - nonlinear

- For x<1
  - Yield stress:
    Below yield stress the strain rate tends to zero at long times
  - Aging decreases transient?
    - I find this very weird
    - Viscosity v. time would be interesting
- For x>1
  - Would be interesting to see viscosity v. time
  - Then we could see more general “thixotropy”?





Discussion

- Does it actually match experimental data?
- Applicability
  - ❑ Predicting behavior for engineering applications?
  - ❑ Better understanding of the physics?
    - Hans Wyss (Harvard) may be interested in nonlinear oscillatory response, to see if it's consistent with his model for SGMs (though this paper didn't discuss it)
  - ❑ If you cite it people may think you're smart

- **SGM** may refer to: (from Wikipedia)
  - ❑ Standard Gross Margin used for classification of agricultural businesses
  - ❑ SG-43 Gorunov machine gun, modified
  - ❑ Second Generation Multiplex DNA profiling system - also see SGM+
  - ❑ Sleepytime Gorilla Museum, an avant-prog band based in Oakland, CA
  - ❑ Soft Glassy Material, a material with structural disorder and metastability (e.g. wet foam, emulsion, paste, or slurry)
  - ❑ Sergeant-Major, a military rank