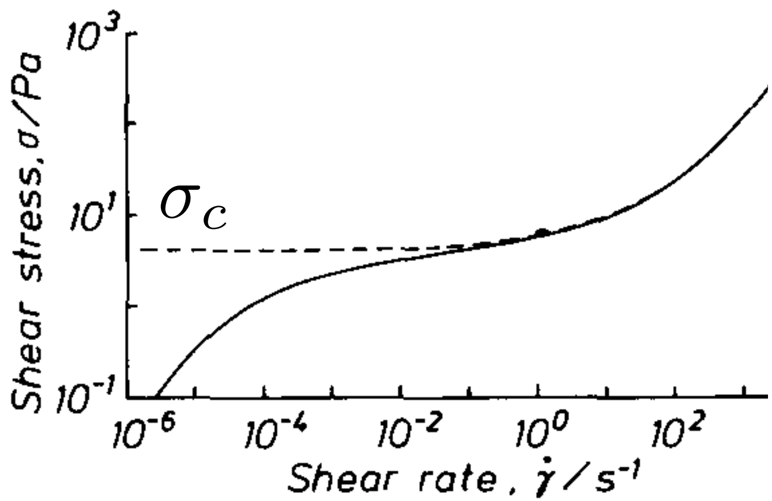
Discussion of “Rutgers-Delaware rule”

Refs:

1. D. Doraiswamy, *et.al.*, JOR (1991).
2. I. Krieger, JOR (1992).
3. H. Barnes, *et.al.*, “Introduction to Rheology” (1989).

Preliminaries

Yield-stress fluid



Herschel-Bulkley model: $\sigma = \sigma_c + K|\dot{\gamma}|^n$

Bingham model: For $n = 1$, $\sigma = \sigma_c + \eta_p\dot{\gamma}$

Symbols

$\gamma(t)$: The "plate strain"

γ_0: Strain amplitude $[\gamma(t) = \gamma_0 cos(\omega t)]$

γ_c: The *critical* strain

$\gamma_r(t)$: The *recoverable/elastic* strain

Doraiswamy model for yield stress fluid

- Below critical yield stress ("Hookean elastic solid"):

$$\sigma = G\gamma_r$$

$$\text{with } \gamma_r(t) \equiv \int_0^t \dot{\gamma}(t')dt' \qquad |\gamma_r| < \gamma_c$$

$$|\gamma_r(t)| = \gamma_c \qquad |\gamma_r| = \gamma_c$$

- Above critical yield stress ("Herschel-Bulkley liquid"):

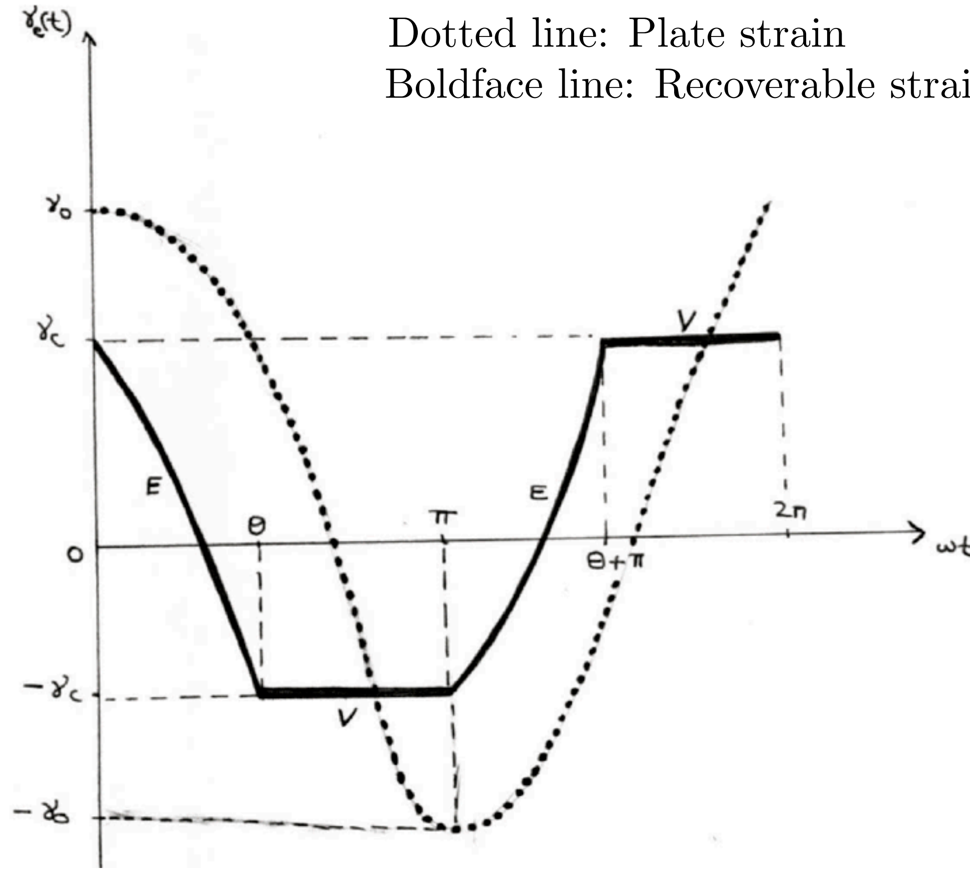
$$\sigma = G\gamma_c + K|\dot{\gamma}|^n \qquad \gamma > \gamma_c = |\gamma_r| \quad (0 < n \leq 1)$$

Note: σ is discontinuous at $|\gamma_r| = \gamma_c$

Recoverable strain

Case: $|\gamma_0| > \gamma_c$

Dotted line: Plate strain
Boldface line: Recoverable strain



For $\gamma(t) = \gamma_0 cos(\omega t)$,

$$\gamma_r(t) = \gamma_c + \int_0^t \dot{\gamma}(t')dt' \implies \gamma_c - \gamma_0(1 - cos\theta) = -\gamma_c$$

Get $\theta = cos^{-1}(1 - \frac{2\gamma_c}{\gamma_0})$, $\quad \theta \in [0, \pi]$ $\quad$ (θ is independent of ω)

Oscillatory Shear

Plan: Calculate complex viscosity and compare with shear viscosity

$$\gamma(t) = \gamma_0 sin(\omega t) \qquad (|\gamma_0| \gg \gamma_c)$$

$$\sigma(t;\omega) = \gamma_0 \omega [\eta'(\omega) cos(\omega t) + \eta''(\omega) sin(\omega t)]$$

$$\eta'(\omega) = \frac{1}{\pi \gamma_0} \int_0^{2\pi/\omega} \sigma(t) cos(\omega t) dt$$

$$\eta''(\omega) = \frac{1}{\pi \gamma_0} \int_0^{2\pi/\omega} \sigma(t) sin(\omega t) dt$$

$$|\eta^*| = \sqrt{\eta'^2 + \eta''^2}$$

Oscillatory Shear

Asymptotic complex viscosity $(|\gamma_0| \gg \gamma_c)$

- For $\omega \to 0$: $|\eta^*| \approx \dfrac{G\gamma_c}{(\gamma_0 \omega)}$
- For $\omega \to \infty$: $|\eta^*| \approx K(\gamma_0 \omega)^{n-1}$

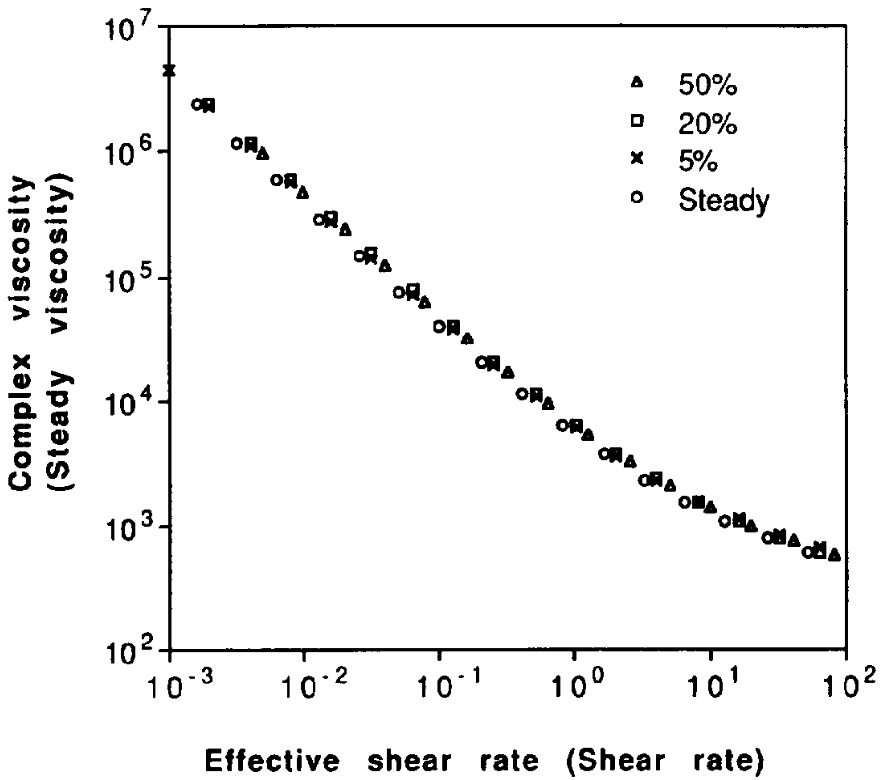
Compare with simple shear:

$$\eta(\dot{\gamma}) \approx \frac{G\gamma_c}{|\dot{\gamma}|} \qquad (|\dot{\gamma}| \to 0)$$

$$\eta(\dot{\gamma}) \approx K|\dot{\gamma}|^{n-1} \qquad (|\dot{\gamma}| \to \infty)$$

Rutgers-Delaware (RD) rule

For materials with a yield stress and a recoverable strain below the yield stress: $\boxed{\eta(\gamma_0\omega) = |\eta^*(\gamma_0\omega)|}$ with γ_0 <u>nonlinear</u>



[70% by volume, 8u silicon particles in LLDPE, CP25]

Krieger (1992)

Assume a structural recovery time τ_r for the material. m is the (odd positive integer) order of the harmonic.

• The RD rule applies to any system whose structural recovery time is longer than the period of oscillation. Despite being highly nonlinear, the stress response should be sinusoidal over the entire frequency range where the RD rule prevails.

Case: $|\gamma_0| \gg \gamma_c,\ 2\pi/(m\omega\tau_r) \ll 1$ Negligible higher harmonic content !

• The RD rule breaks down at low frequencies, where significant recovery can take place during a cycle. At low frequencies, odd harmonics should appear in the stress response, which will therefore be non-sinusoidal.

Case: $|\gamma_0| \gg \gamma_c,\ 2\pi/(m\omega\tau_r) \gg 1$ Non-asymptotic, model-dependent !